The Speed of Light and Time Dilation – A Quandary

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It is a well-known fact of relativity that time slows down for things that move close to the speed of light. This concept, although somewhat strange to those of us who live at non-relativistic speeds, is actually easy to understand. However, it leads to a logical paradox that I have struggled to understand for more than a decade. Undoubtedly, the theory (and reality!) are fine, and it is my understanding that is faulty. This essay will try to explain the problem that has vexed me for so long. Perhaps someone smarter than I can resolve the apparent paradox.

Time Dilation Background

The most common example given of the “Time Dilation” phenomenon is the case of an astronaut who leaves earth at very high speed for a distant star. To the astronaut, the journey there and back takes only a few decades, but upon his return, he discovers to his horror that millions of years have passed on earth, and everyone he knows is long dead. (Apparently, this astronaut never heard of Einstein and slept through his physics classes in college.)

This time dilation effect has been experimentally confirmed in many cases. Short-lived sub atomic particles last longer when they are accelerated to nearly the speed of light. Even the clocks on board the space shuttle are noticeably (if just barely) out of synchronization with ground clocks at the end of a mission due to its high orbital speed. The high speed of GPS satellites throws off their internal clocks and has to be factored into the calculations performed by our GPS receivers, or they would give us incorrect location information.

Albert Einstein postulated this effect as a direct result of his theory of special relativity, which required that the speed of light be constant in all frames of reference. Einstein was no math whiz, so he used a very simple “thought experiment” to demonstrate the effect. This thought experiment is so simple that even a layman can easily understand the effect with just a few minutes consideration.

There is one very troubling aspect of this effect that I cannot fathom, however. Why does the astronaut stay young and the earth get old? Since there is no absolute frame of reference in the universe, speed is always measured relative to other objects. You could just as easily say the earth speeds away and the astronaut stays motionless. Why doesn't the astronaut get old instead of the earth? Why does one age, and not the other? It seems arbitrary to me.

This leaves me with the quandary: "**Why doesn't the other clock slow down instead?**"

The Famous Thought Experiment

Einstein imagined an experimenter on a train who has 2 horizontal mirrors, one on the floor, and the other on the ceiling. He shines a light between the 2 mirrors and uses a very precise stopwatch to determine how long it takes for the light to bounce off the mirror on the ceiling and hit the mirror on the floor. (See Figure 1 – Train at Rest)

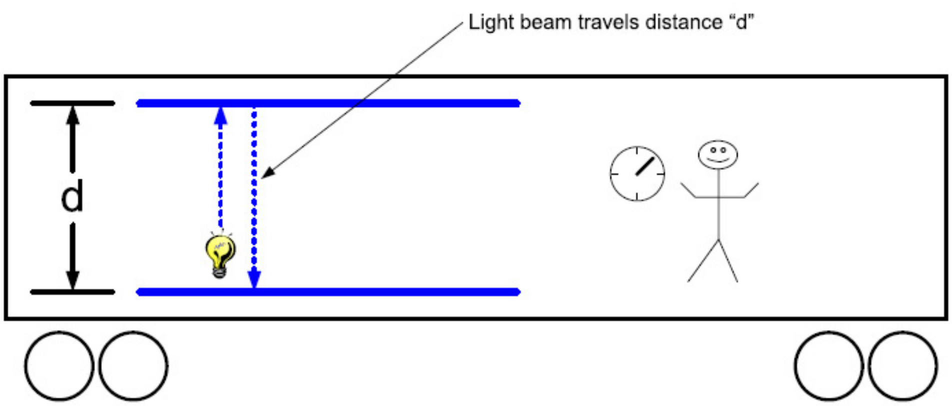

Figure 1 Train at Rest

The time can be predicted if we know the distance between the mirrors ("**d**"), by using the formula:

$$\textbf{Speed} = \frac{\textbf{Distance}}{\textbf{Time}}$$

We can solve for time by using the equation:

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

Einstein knew, based partly on the famous Michelson-Morley experiments (which were performed at my alma mater in Cleveland in 1887), that the Speed of Light has to be the same, no matter where you are or how you measure it. Since the speed of light (which we call "*c*", or approximately 186,000 miles per second) is known, and we know the distance "**d**", it is easy to calculate the time, and to confirm that measurement with a handy stopwatch.

The interesting thing happens when the train starts to move. (see Figure 2 – Train in Motion) To the experimenter riding in the train, everything is as it was when the train was at rest. However, if you were to be standing on a station platform and watching this experiment as the train whizzed by, you would see things a little differently. The light beam would not simply move up and down, but would seem (to you) to follow a diagonal path as the train moved. The light beam now doesn't travel a distance **d**, but instead has to travel a distance $\mathbf{d_M}$, which is longer than **d**.

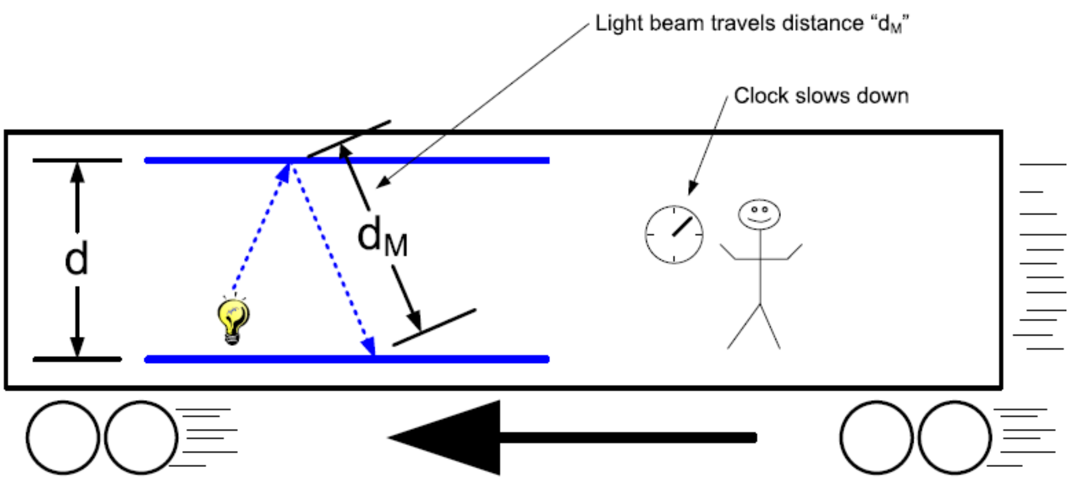

Figure 2 Train in Motion

Now comes the weird part: Since the speed of light is constant in both frames of reference (the platform and the speeding railcar), the ratio of Distance/Time has to remain the same. However, since the distance $\mathbf{d_M}$ is longer than **d**, that means the amount of time has to shrink proportionately to keep everything in balance.

Remember our general equation:

$$\textbf{Speed} = \frac{\textbf{Distance}}{\textbf{Time}}$$

In this case, “speed” is equal to “c”, the speed of light. Also “distance” is either **d** or $\mathbf{d_M}$ depending on your frame of reference. So the following equation holds:

$$\textbf{Speed} = \frac{\textbf{Distance}}{\textbf{Time}} = c = \frac{\textbf{d}}{\textbf{Time}} = \frac{\mathbf{d_M}}{\mathbf{Time_M}}$$

Using a bit of algebra gets us to this equation:

$$(\ \mathbf{d}\ )(\ \mathbf{Time_M}\ ) = (\mathbf{d_M}\ )(\ \mathbf{Time}\ )$$

Therefore:

If:

$$\mathbf{d_M > d}$$

then:

$$\mathbf{Time_M > Time}$$

Ergo, our clocks run slower when in motion. The amount they run slower is directly proportional to the difference in the lengths of **d** and $\mathbf{d_M}$.

Both Cars in Motion

So the clocks slowing is a little weird, but it is understandable, once one assumes that speed of light is constant in all frames of motion. The problem for me is that ***speed*** is always measured relative to something else. My car can travel at 65 miles per hour, relative to the surface of the earth. The space shuttle hurtles around the earth at approx 16,000 miles per hour, relative to any one point on earth. In Einstein’s thought experiment, the train rushed past the observer standing on the station platform at something “close” to 186,000 miles per second.

But what if the observer was standing on a railroad car that was on a parallel siding? What if it was dark, and the observer could not tell if it was his car rushing at this high speed, or the other car? (See Figure 3 – passing Trains) Since the only measurable speed in this case would be the speed of the cars relative to each other, it should not matter which car was in motion. [Note, we will assume that both trains have superb suspensions and so neither rider can feel the vibration of the track.]

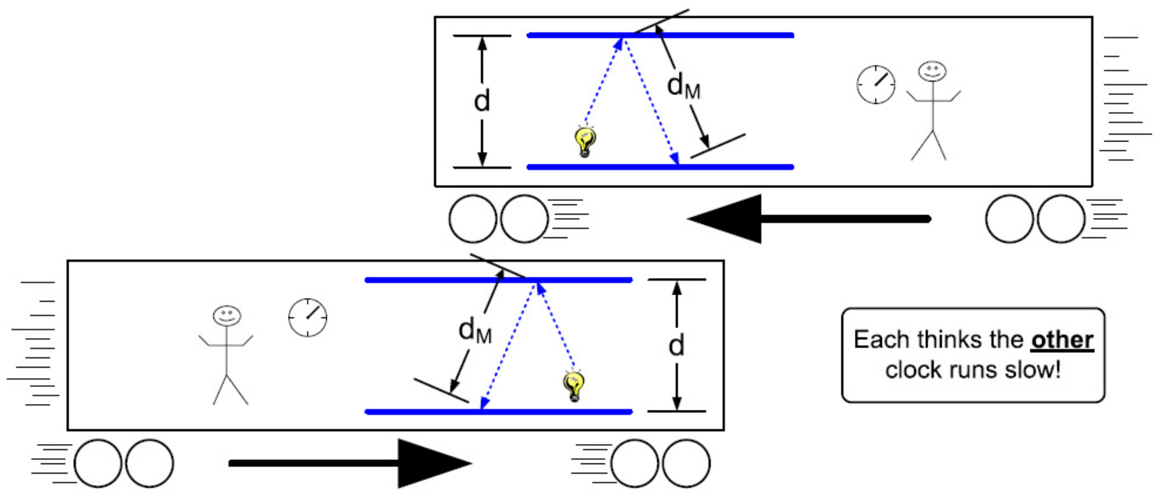

Figure 3 Passing Trains

By the best of my (very limited) ability to reason, **both** observers should think the other guy's clock is running slow!

Complicating the Thought Experiment

OK, I could sorta handle this, since everybody says that 'simultaneity" is no longer a valid concept when one reaches relativistic speeds. But now let's add something else to the thought experiment: Each person in the 2 cars on the parallel tracks announces on the radio what their local time is. As they speed closer to each other (neither knowing which car is actually moving nor which is actually stationary), they continually broadcast their own local time. (see Figure 4 – Synchronize Clocks) In this case, even though each rider ought to think the other guy's clock is running slow, the factual data of the actual transmitted time signal would seem to be in conflict with the "slow clock" data.

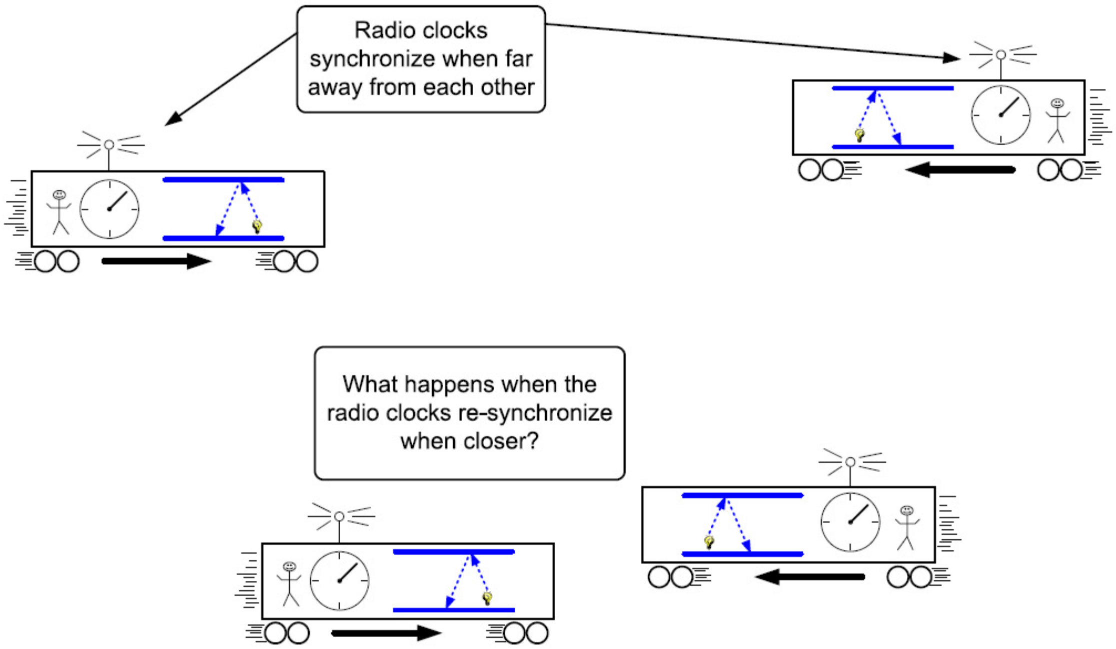

Figure 4 Synchronize Clocks

If each observer on each car thinks the other guy's clock is running slow, what will happen when they receive the updated time signal? Will each rider have his own "bubble" of local time with the other's clock always reading slower? Both clocks can't actually be running slower than the other guy's, can they?

My Quandary

What is the solution to this paradox? It is generally assumed that there are no paradoxes in nature, and if one is found, it indicates that an error exists in one's analysis. So is this a defect in the thought experiment, in my reasoning, or in my assumptions? I don't have any idea!